MIT | Arts, Commerce & Science College

Department of Computer Science
Class: S.Y.B.Sc. (Comp. Sci.)
Mathematics Question Bank

Sub: Applied Algebra

Ch. 1: General Vector Spaces

2 Marks

1) Determine the values of 'λ' so that the set $\{(\lambda,2),\ (8,\lambda)\}$ is L.I.
2) Give an example to illustrate that union of two subspaces of a vector space need not be subspace.
3) Does there exists a subspace W of $\mathbb{R}^3$ such that dimension of W is 4. Justify your answer.
4) State if the following statement is True *or* False & justify your answer:
   "The row vectors of a matrix of order 20×10 are linearly independent."
5) State whether the following statement is true *or* false & justify :
   "There exists a matrix of order 5×3 whose row rank is 4."
6) Consider a basis $B=\{1+2x,-x\}$ for a vector space P_1. Find the coordinate vector of $2+7x$ with respect to the basis B.
7) Find the value of a real number 'α' such that $\overline{v}=(1,-2,\alpha)$ is a linear combination of: $\overline{u}=(3,0,-2)$ and $\overline{w}=(2,-1,-5)$.
8) Consider a basis $B=\{1+x,-2+x\}$ for a vector space P_2. Find the vector whose coordinates with respect to basis B are 4 and 1.

5 Marks

9) Show that intersection of two subspaces of a vector space is a subspace. Is union of two subspaces a subspace? Justify.
10) Let $V=\mathbb{R}^+$ be the set all positive reals. Define addition of any two members $\overline{x}$ and $\overline{y}$ to be the usual multiplication of numbers that is $\overline{x}+\overline{y}=xy$, define scalar multiplication by a scalar k to any $\overline{x}\in\mathbb{R}^+$ to be x^k that is $k\overline{x}=x^k$ then determine whether V is a vector space.
11) Determine whether the following vectors span $\mathbb{R}^3$ $v_1=(1,2,3),\ v_1=(0,0,1),\ v_1=(0,1,2)$.
12) Determine if the set $\{(3,1,-4),\ (2,5,6),\ (1,4,8)\}$ of vectors forms a basis for $\mathbb{R}^3$.

13) Find the basis for null space of the following matrix: $\begin{bmatrix} 1 & 2 & 3 & 0 \\ -1 & 1 & 1 & 4 \\ 3 & 6 & 9 & 0 \end{bmatrix}$ Hence find nullity and rank of the given matrix.

14) Determine whether the following subset W is a subspace of the vector space $M_{2\times2}$: $W = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \middle/ ad - bc = 0,\ a,b,c,d \in \mathbb{R} \right\}$.

15) Let $S = \left\{ \overline{u_1}, \overline{u_2}, \ldots\ldots, \overline{u_r} \right\}$ be a set of vectors in $\mathbb{R}^n$. If $r > n$, then prove that S is linearly dependent.

16) Find the basis of the subspace of $\mathbb{R}^5$ spanned by the vectors $\overline{u} = (1,-1,0,1,2)$, $\overline{v} = (0,2,-1,3,0)$, $\overline{w} = (-1,5,-2,5-2)$ and $\overline{z} = (1,-3,1,-2,2)$.

17) Find the coordinate vector of $\overline{u} = (9,7,5)$ relative to the basis $S = \{(0,0,1),\ (0,2,2),\ (3,3,3)\}$ of $\mathbb{R}^3$.

18) Determine the value of 'a' for which the following set of vectors in $\mathbb{R}^3$ is linearly dependent: $\{(a,-2,-2),\ (-2,a,-2),\ (-2,-2,a)\}$.

19) Determine if the vector $\overline{p} = 1 + x + x^2$ belongs to the linear span of the set: $\left\{1 - x + x^2, 2 + x, x + x^2, 2 + x + x^2\right\}$.

20) Find a basis of the column space of the following matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 1 \\ 2 & 2 & 2 \\ 4 & 1 & -2 \end{bmatrix}$$

Hence find rank of A.

21) Find the basis of the column space of A consisting entirely column vectors of A. where,

$$A = \begin{bmatrix} 1 & 2 & 0 & 2 & 5 \\ -2 & -5 & 1 & -1 & -8 \\ 0 & -3 & 3 & 4 & 1 \\ 3 & 6 & 0 & -7 & 2 \end{bmatrix}$$

22) Find the basis of the row space of A consisting entirely row vectors of A. where,

$$A = \begin{bmatrix} 1 & -2 & 0 & 0 & 3 \\ 2 & -5 & -3 & -2 & 6 \\ 0 & 5 & 15 & 10 & 0 \\ 2 & 6 & 18 & 8 & 6 \end{bmatrix}$$

23) Find rank and nullity of the matrix $A=\begin{bmatrix}1 & -2 & -1\\ 2 & -1 & 3\\ 7 & -8 & 3\\ 5 & -7 & 0\end{bmatrix}$ and verify rank nullity theorem.

Ch. 2: Eigen Values and Eigen Vectors

2 Marks

24) Determine if the vector $\overline{X}=\begin{bmatrix}3\\ -3\\ -3\end{bmatrix}$ is an eigenvector of the matrix: $A=\begin{bmatrix}4 & 0 & 1\\ -2 & 1 & 0\\ -2 & 0 & 1\end{bmatrix}$.

25) If the eigenvalues of the matrix are $1,3$ *and* 5, then find eigenvalues of A^{-1} *and* A^3.

26) If $'k'$ is a positive real number, then what are the eigenvalues of kI_n, where I_n is the identity matrix of order $n\times n$? Is kI_n positive definite matrix? Justify your answer.

27) State whether the following statement is true *or* false with justification:
"There exists a positive definite, symmetric matrix A that has eigenvalues $3, 1$ and -2."

28) Determine if the following statement is true or false & justify your answer.
"If the eigen values of a matrix A are $0,2,2$ then A is invertible."

29) Find the symmetric matrix A such that the quadratic form
$$x^2-4y^2+z^2+4xy-6xz+2yz$$
is expressed in the form X^TAX.

5 Marks

30) Find the eigenvalues and eigenvectors corresponding to the matrix $A=\begin{bmatrix}1 & 0 & -1\\ 1 & 2 & 1\\ 2 & 2 & 3\end{bmatrix}$. Also, find dimension and basis for the eigenspace the matrix.

31) If λ is an eigenvalue of matrix A, then show that $\dfrac{\det(A)}{\lambda}$ is eigenvalue of adjoint of A.

32) Find all eigenvalues of the matrix
$$A=\begin{bmatrix}5 & -6 & -6\\ -1 & 4 & 2\\ 3 & -6 & -4\end{bmatrix}$$
Also find basis of the eigen space of A corresponding to maximum eigenvalues of A.

33) Find all the eigenvalues of the matrix A. Also, find eigenspace corresponding to the smallest eigenvalues: $A=\begin{bmatrix}1 & -3 & 3\\ 3 & -5 & 3\\ 6 & -6 & 4\end{bmatrix}$

34) Find the matrix P (if it exists) that diagonalizes the following matrix: $A=\begin{bmatrix}1 & 4\\ 1 & 1\end{bmatrix}$.

35) Verify Cayley-Hamilton theorem for the following matrix A and hence find A^{-1} : $A=\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}$.

36) Determine whether the matrix $A=\begin{bmatrix}10 & -9\\ 4 & -2\end{bmatrix}$ is diagonalizable. Justify.

37) Consider the matrix $A=\begin{bmatrix}1 & 2\\ 0 & -2\end{bmatrix}$

i) Find all eigen-values of A.
ii) Find the bases for all eigenspaces of A.
iii) Determine if A is diagonalizable. If yes, find the matrix P that diagonalizes A.

38) Find minimum value of the quadratic form: $3x_1^2+3x_2^2+2x_1x_2$ subject to the constraint $x_1^2+x_2^2=1$ & determine the value of x_1 & x_2 at which minimum occur.

39) Find maximum value of the quadratic form: $2x_1^2+2x_2^2+6x_1x_2$ subject to the constraint $x_1^2+x_2^2=1$ & determine the value of x_1 & x_2 at which maximum occur.

Ch. 3: Linear Transformations

<u>2 Marks</u>

40) Consider the linear transformation $T:\mathbb{R}^3\to P_1$, defined by
$$T(a,b,c)=(a+6b-2c)+(2a-4b+c)x.$$
Determine if the vector $\bar{u}=(6,15,48)$ belongs to $\ker T$.

41) Let $T:\mathbb{R}^3\to\mathbb{R}^2$ be defined by
$$T(x,y,z)=(x+2y+z,1).$$
Determine if T is a linear transformation.

42) Define kernel of a linear transformation. Consider the linear transformation: $T:\mathbb{R}^3\to\mathbb{R}^3$, defined by :
$$T(x,y,z)=(2x,x-y+z,3x+2y-2z).$$
Determine if the vector $(0,1,1)$ belongs to $\ker T$.

43) Consider the linear transformation $T:\mathbb{R}^2 \to \mathbb{R}^2$ defined by:

$T(x, y) = (x+y, x-y)$.

Find the standard matrix of T.

44) Let $T:\mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation defined by :

$$T(x, y) = (4x+y, -x+3y),$$

Find standard matrix of T.

45) Let $T: P_2 \to \mathbb{R}^4$ be a linear transformation such that $rank(T)=1$. Find $nullity\ of\ T$.

5 Marks

46) If $\left(\overline{e_1}, \overline{e_2}, \overline{e_3}\right)$ is standard basis of $\mathbb{R}^3$ and if $T:\mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation such that,

$$T\left(\overline{e_3}\right) = 2\overline{e_1} + 3\overline{e_2} + 5\overline{e_3}$$
$$T\left(\overline{e_2} + \overline{e_3}\right) = \overline{e_1}$$
$$T\left(\overline{e_1} + \overline{e_2} + \overline{e_3}\right) = \overline{e_2} - \overline{e_3}.$$

Compute $T\left(\overline{e_1} + 2\overline{e_2} + 3\overline{e_3}\right)$.

47) Suppose $T:\mathbb{R}^3 \to \mathbb{R}^2$ is a linear transformation defined as

$$T(x_1, x_2, x_3) = (x_1 + x_2 + x_3, -x_2 + x_3).$$

Find kernel and range of T.

48) Suppose: $A = \begin{bmatrix} 2 & 4 \\ 2 & 0 \\ 7 & -3 \end{bmatrix}$ is the matrix of the linear transformation: $T: P_1 \to P_2$ with respect to the bases $B = \left\{\overline{p_1}, \overline{p_2}\right\}$ and $B' = \left\{\overline{q_1}, \overline{q_2}, \overline{q_3}\right\}$ where: $\overline{p_1} = 1+x,\ \overline{p_2} = 1-x,\ \overline{q_1} = 1,\ \overline{q_2} = x$ and $\overline{q_3} = x^2$.

a) Find coordinate matrices $\left[T\left(\overline{p_1}\right)\right]_{B'}, \left[T\left(\overline{p_2}\right)\right]_{B'}$.

b) Find $T\left(\overline{p_1}\right)$ and $T\left(\overline{p_2}\right)$.

c) Use the matrix A to compute $T(3-7x)$.

49) Consider the linear transformation: $T: P_2 \to \mathbb{R}^3$, such that:

$$T(1) = (2,1,0),\ T(x) = (-1,3,1) \text{ and } T\left(x^2\right) = (4,2,5).$$

Find the formula for $T\left(a+bx+cx^2\right)$. Use this formula to compute $T\left(1+2x+3x^2\right)$.

50) If $T: V \to W$ is a linear transformation, then prove that $Ker\,T$ is a subspace of V.

51) If $T: U \to V$ is a linear transformation, then show that $range(T)$ is a subspace of V.

52) Let $T: M_{2\times 2} \to \mathbb{R}^3$ be a linear transformation defined by:

$$T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = (2a-b+d,\ -b-c-d,\ 3a+d).$$

Find matrix of T w.r.t. the basis $B = \{A_1, A_2, A_3, A_4\}$ of $M_{2\times 2}$ and B' being the standard basis for $\mathbb{R}^3$, where:

$$A_1 = \begin{bmatrix} 3 & 6 \\ 3 & -6 \end{bmatrix},\ A_2 = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix},\ A_3 = \begin{bmatrix} 0 & -8 \\ -12 & -4 \end{bmatrix},\ A_4 = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}.$$

53) Consider the linear transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$, defined by

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x-2y \\ -3x+6y \end{bmatrix}.$$

Find a basis of range of T and hence find $rank(T)$.

54) Let $T: \mathbb{R}^3 \to \mathbb{R}^4$ be a linear transformation defined by :

$$T(a,b,c) = (a+2b+3c, -2a-b, a-2b-5c, 4b+8c).$$

Find a basis for kernel of T and hence find rank of T.

55) Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be defined by :

$$T(x,y,z) = (x+y, y+z, z+x).$$

Show that T is linear isomorphism. Find similar formula for T^{-1}.

56) Suppose $T: P_2 \to \mathbb{R}^2$ be a linear transformation, defined by:

$$T(a+bx+cx^2) = (a+2b+c, a-b-c).$$

a) Find the matrix of T with respect to the bases $B = \{\overline{p_1}, \overline{p_2}, \overline{p_3}\}$ for P_2 and $B' = \{\overline{e_1}, \overline{e_2}\}$ for $\mathbb{R}^2$; where $\overline{p_1} = 1-x-x^2$, $\overline{p_2} = 2+x^2$, $\overline{p_3} = 1$ and $\overline{e_1} = (1,0)$, $\overline{e_2} = (0,1)$.

b) Use the matrix obtained to compute $T(\overline{p})$, where $\overline{p} = 7-2x+x^2$.

Ch. 4: Groups and Coding

2 Marks

57) Find the number of generators of a cyclic group of order 3.

58) Define hamming distance between two code words. Also find the Hamming distance between x and y; where $x = 110110110$ and $y = 010110111$.

59) Consider the $(6,7)$ parity check code. For both of the following received words, determine whether an error will be detected:

a) 1010011.

b) 1010100.

60) If 'e' is $(2,6)$ encoding function with minimum distance 4, then how many errors can 'e' detect? How many errors can corrected?

5 Marks

61) Let $G=(\mathbb{Z}_4,+)$ and $H=\{\bar{0},\ \bar{2}\}$. Write all the cosets of G related to its subgroup H.

62) Let $H=\begin{bmatrix}0&1&1\\0&1&1\\1&0&0\\0&1&0\\0&0&1\end{bmatrix}$ be a parity check matrix. Determine $(2,5)$ group code function $e_H: B^2 \to B^5$.

63) Let $H=\begin{bmatrix}1&1&0\\0&1&1\\1&0&0\\0&1&0\\0&0&1\end{bmatrix}$ be a parity check matrix. Determine $(2,5)$ group code function $e_H: B^2 \to B^5$.

64) Suppose $e: B^m \to B^n$ is a group code. Write the procedure for obtaining a maximum likelihood decoding function associated with 'e'.

65) Consider the $(2,5)-$ group encoding function, defined by:

s $$\begin{aligned}&e(00)=00000,\ e(01)=01101,\\&e(10)=10011,\ e(11)=11110.\end{aligned}$$.

Decode the word '01100' relative to a maximum likelihood decoding function.

66) In RSA cryptosystem take $p=79,\ q=89$ and $d=119$. Determine the public key (e,n).

67) In RSA cryptosystem take $p=61,\ q=53$ and $e=17$. Determine the private key (d,n).

Sub: Numerical Techniques

Ch.1: Errors

2 marks

1) Round off the number 0.005998 to the decimal digits, find it's relative & percentage error.
2) Define relative & percentage error.
3) Round off 0.848586 correct to 4 significant figures & find absolute error.
4) Round off 0.456789 correct to 4 significant figures & find relative error.
5) Define Percentage error & absolute error.
6) If the approximate value of 2013 is 2, find percentage error.
7) Round off 1.53364 correct to 3 significant figures & find percentage error.
8) If $y=\operatorname{cosec}^2 x$, then find error in x.

Ch.2: Algebraic & Transcendental Equations

2 marks

1) Using Newton-Raphson method, obtain the formula to find out cube root of 'p'
2) Write the formula for Regular- Falsi method.
3) State True of False: Rate of convergence of Newton-Raphson method is 2.
4) Find the approximate root of the equation $x^3-2x-5=0$ in(2,3) by using Regula Falsi method.(Perform 1 iteration)
5) Compute square root of 57 by using Newton-Raphson method.Take $x_0=7.5$.(Perform 2 iteration)
6) Write the formula for Newton-Raphson formula for square root of any real number.
7) Find the root of the equation $x^3+x+1=0$ using Newton Raphson method.
8) Compute $\sqrt{27}$ by using Newton-Raphson method.

5 marks

1) Find real root of the equation $x\sin x+\cos x=0$ correct to three decimal places using Newton-Raphson method.
2) Use Regular Falsi method to find approximate root of the equation $x^3+x-1=0$ in the interval [0,1] correct up to 2 decimal places.
3) Obtain the real root of the equation $x^3-9x+1=0$ by Regula Falsi method correct up to three decimal.
4) Obtain the real root of the equation $x^3-2x-5=0$ by Regula Falsi method correct up to two decimal.

5) Find the real root of the equation $\log x - \cos x = 0$ correct to three decimal places using Newton Raphson method.(Take x_0=1.3)
6) Find $\sqrt[3]{18}$ by Newton Raphson method. Perform 4 iterations.(Take x_0=25)

Ch.3 Calculus of Finite Differences

2 marks

1) Evaluate $\Delta^2(a^{5x-7})$.(Take h=1)
2) Prove that $E^{-1} = 1-\Delta$.
3) State True or False: Every equation of nth degree has only (n-1) roots. Justify.
4) Evaluate $\Delta(\cot^{-1} x)$.
5) Write he forward deference table.

x	1	3	5	7	9
y	2	10	26	50	82

6) Prove that: $\Delta\nabla = \Delta - \nabla$
7) Evaluate: $\Delta^2\left[3^{4x+5}\right]$.
8) Show that $\delta E^{1/2} = \Delta$
9) Write the formula for d^2y/dx^2 using Newton's forward difference formula for non tabular values of x.
10) State fundamental theorem for difference calculus and hence evaluate,
$\Delta^{10}\left[(1-x)(1-2x^2)(1-3x^2)(1-4x^2)\right]$.(Take h=1)
11) Define averaging operator μ

5 marks

1) State & prove Fundamental theorem on difference of polynomial.
2) Estimate the missing term in the following.

x	1	2	3	4	5	6	7
y	2	4	8	?	32	64	128

3) Estimate the production for the year 1964 & 1966 from the given data.

Year	1961	1962	1963	1964	1965	1966	1967
Production	200	220	260	?	350	?	430

4) Find the error & correct the wring figure in the following data:

x	0	1	2	3	4	5	6
y	4	7	12	20	28	39	52

Ch.4 Interpolation with Equal Interval

2 Marks

1) Write the formula for dy/dx using Newton's backward Interpolation formula.
2) Write the formula for dy/dx using Newton's forward Interpolation formula.
3) State Gauss forward Difference Interpolation formula.
4) State Gauss Backward Difference Interpolation formula

5 Marks

1) Find f(4.4) from the following.

x	0	2	4	6	8	10	12
y	12	7	6	7	13	32	77

2) Find the number of students who obtained less than 45 maks.

Marks	30-40	40-50	50-60	60-70	70-80
No. of students	31	42	51	35	31

3) If $u_{75} = 246 , u_{80} = 202 , u_{85} = 118 , u_{90} = 40 , find\ u_{79.}$
4) Derive Newton Gregory formula for Forward Difference Interpolation.
5) State & prove Newton's backward interpolation formula for equally spaced points.
6) The populations on a town in decimal census were as under.

Year	1921	1931	1941	1951	1961
population (thousands)	46	66	81	93	101

7) Using Gauss backward formula,estimate the value of f(5.8) from the following deta.

x	4	5	6	7
F(x)	270	648	1330	2448

8) Using Gauss Forward formula, find a polynomial which takes the values of y as given in the table. What is the degree of Interpolating polynomial?

x	2	4	6	8	10
y	-2	1	3	8	20

9) From the following values obtain the value of f(25) using Bessel's formula.

x	F(x)
20	2854
24	3162
28	3544
32	3992

Ch. 5 Interpolation with Unequal Interval

2 Marks

1) Write Lagrange's Interpolation formula for unequal interval.
2) State Hermities Interpolation formula.
3) State Newton's divided difference formula.
4) Define Divided Differences of 1st and 2nd order.
5) Find the divided differences for the function $f(x) = x^2 + 2x + 2$ with arguments 1, 2, 4,7,10.

5 Marks

1) Find a polynomial f(x) for the following data also find f(2).

x	0	1	3	4
y	-12	0	6	12

2) Express the function $f(x) = \dfrac{x^2 + 6x + 1}{(x-1)(x+1)(x-4)(x-6)}$ as sums of partial fractions by using Lagrange's Interpolation formula.
3) State and prove Newton's divided difference formula.
4) Find a polynomial of lowest degree which passes through the points (3,3),(2,12),(1,15),(-1,-21).
5) Apply Hermities Interpolation formula to obtain 4th degree polynomial from the data given below.

x_i	0	1	2
y_i	1	0	9
Y_i'	0	0	24

6) Use Lagrange's formula to find the value of y when x=10,if the values are given by

X	5	6	9	11
y	12	13	14	16

Ch. 6 Numerical Integration

2 Marks

1) If f(0)=0,f(1)=2,f(2)=6,f(3)=12,evaluate $\int_0^3 f(x)\,dx$ by Simpson's 3/8th rule.
2) If f(0)=0,f(1)=2,f(2)=6,f(3)=12 and f(4)=18,evaluate $\int_0^4 f(x)\,dx$ by Simpson's 1/3th rule.
3) Write the general quadrature formula for Numerical Integration.
4) Write Trapezoidal rule for Numerical Integration.

5) Evaluate $\int_0^1 x^2$ by Trapezoidal rule. Take h=0.5
6) State Euler-Maclarin's formula for Numerical Integration.

5 Marks

1) State & derive Simpson's (3/8th) rule for Numerical Integration.
2) Evaluate the integral $I = \int_3^5 \frac{4}{2+x^2} dx$,using Simpson's 1/3rd rule.(Take h=0.25)
3) Derive Quadrature formula for Numerical integration.
4) A solid of revolution is formed by rotating about x axis, the area between x axis, the line x=0 & x=1 and a curve through the points with the coordinates.

x	0.00	0.25	0.50	0.75	1
y	1.0000	0.9896	0.9589	0.9089	0.8415

5) State & derive Simpson's (1/3th)rule for Numerical Integration.
6) Evaluate: $\int_0^1 e^x dx$ by dividing [0,1] into 10 equal parts by using Simpson's 1/3rd rule.

Ch. 7 Numerical solution of Ordinary Differential Equation

2 Marks

1) Write Runge-Kutta formula of second order for solving ordinary differential equation.
2) Given $\frac{dy}{dx} + 2y = 0 \,; y(0) = 1,$ find y(0.1) using Euler's method.
3) Given that $\frac{dy}{dx} = 1 + xy$ with y (0)=1.Find y(0.1) by Euler's method.
4) Given: $\frac{dy}{dx} = 1$ with y(0)=0.Find y(0.1) by Euler method.

5 Marks

1) Derive the formula to solve $\frac{dy}{dx} = f(x, y)$ where y=y_0 when x=x_0 by Euler's method.
2) Determine the value of y(0.1) up to 4 decimal places using Euler's modified method where $\frac{dy}{dx} = x + y \,,\; y(0) = 1 \; and \; h = 0.05$
3) Determine the value of y when=0.1 by using Euler' modified method. where $\frac{dy}{dx} = x^2 + y \,,\; y(0) = 0.94$

4) Derive the formula for Euler's modified method hence use it to find the value of y(0.1) correct up to 4 decimal places, where $\frac{dy}{dx} = 1 + xy\ ; y(0) = 2.(Take\ h = 0.1)$
5) Given that $\frac{dy}{dx} = x + y^2$ with y(0)=1.Find y(0.1) by Runge-Kutta method of fourth order.
6) Determine the value of y(0.1) correct up to 3 decimal places using Euler's modified method where $\frac{dy}{dx} = y - \frac{2x}{y}\ , y(0) = 1$
7) Solve $\frac{dy}{dx} = -2xy^2$ with x=2.1 using Runge's Kutta fourth order method to find y (0.2).

<u>10 Marks</u>

1) Given that: $\frac{dy}{dx} = y - x$ with y(0)=2.Find y(0.1) and y(0.2) by Runge-Kutta method of fourth order.
2) Given that y'=x+y with y(0)=1.Find y(0.1) and y(0.2) by using Runge-Kutta method of fourth order.

Sub: Computational Geometry

Ch.1 2D-TRANSFORMATION

2Marks

1. Write the transformation matrix for shearing in x and y directions by -2 and 5 units respectively. Apply it on the point P[3, 4].
2. A circle with circumference 8π cm is uniformly scaled by 3 units. Find the area of transformed figure.
3. Find the point at infinity on the line $2x+3y=5$
4. If the line passing through two points A[2, 3] and B[-4, 7] is transformed under:
   $$[T]=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$$
   Then find the slope of the resulting line.
5. Write the transformation matrix for translation in x and z directions by -3 and 5 units respectively.
6. If the line segment joining A[2, 9] and B[-4, 3] is reflected through the line Y=0, then find the midpoint of the transformed line segment.
7. Determine whether the transformation matrix:
   $$[T]=\begin{bmatrix}1 & 1\\ 1 & 2\end{bmatrix}$$
   represents a solid body transformation. Justify?
8. Write a 2x2 transformation matrix for rotation about the origin through 45°.
9. If we apply 2x2 transformation matrix $[T]=\begin{bmatrix}1 & 3\\ 2 & 4\end{bmatrix}$ on the line $X-5Y=10$, then find the slope of the resulting line.
10. True or false: "Reflection is a solid body transformation." Justify?
11. Reflect the point P[1, 2] through the line X=0.
12. What is the effect of the transformation matrix $[T]=\begin{bmatrix}1 & -5\\ 0 & 1\end{bmatrix}$ on a two dimensional object?
13. If line L is transformed to the line L* using a transformation matrix $[T]=\begin{bmatrix}2 & 3\\ 4 & 5\end{bmatrix}$ and slope of L* is ½, find the slope of the line L.
14. Find the transformation matrix to translate the point P[3, -5] so that it coincides with the origin.

15. The triangle whose area is 4sq.units is transformed by using transformation matrix $[T]=\begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$. Obtain the area of transformed figure.
16. Determine if the transformation of reflection through Y – axis is a solid body transformation. Justify your answer?
17. The ΔABC with position vectors A [1, 0], B [0, 1] and C [-1, 0] is transformed by $[T]=\begin{bmatrix} 4 & 3 \\ -1 & 2 \end{bmatrix}$ to new $\Delta A^{'}B^{'}C^{'}$. Find the area of new $\Delta A^{'}B^{'}C^{'}$.
18. Write the 2x2 transformation matrix for reflection through the line y = -x.
19. True or false: "Reflection through X – axis is a solid body transformation". Justify your answer.
20. First reflect the point P [10.5, 7.3] through x – axis and then translate in x and y directions by -6.5 and 15.7 units respectively. Find the transformation matrix.
21. If we apply shearing in x and y directions by 2 and -3 units respectively, onto a circle, then we get a plane figure of area $(343\pi)cm^2$. Determine the radius of the original circle.
22. If we apply the 2x2 transformation matrix onto the points A[1, 0] and B[0, 1] then they are transformed to the points A*[4, 1] and B*[-2, 2] respectively. What is the transformation matrix?
23. Find the angle of rotation θ, so that the line y = -x coincides with X-axis.
24. The line segment joining A [5, 6] and B [-4, -4] is scaled uniformly by factor 8 units. Find the midpoint of the transformed line segment.
25. What is meant by solid body transformation?
26. What are the conditions for a solid body transformation?
27. What is the determinant of inverse of any pure rotation matrix?

5 Marks

1. Prove that the midpoint of the line segment AB is transformed to the midpoint of segment A'B' under 2x2 transformation matrix [T].
2. If the line segment joining the points A[2, 4] and [12, 16] is transformed under $[T]=\begin{bmatrix} 3 & 1 \\ -2 & 1 \end{bmatrix}$, then find the midpoint of the resulting line segment $A^{*}B^{*}$.
3. If the two lines $x+2y=2$ and $x+y=4$ are transformed under the transformation matrix $[T]=\begin{bmatrix} 2 & -1 \\ -2 & 1 \end{bmatrix}$, Then find the point of intersection of resulting lines.

4. If the intersection of two lines $4x+5y=13,\ x+2y=4$ is transformed under transformation matrix $[T]=\begin{bmatrix}1 & 2\\ -1 & 2\end{bmatrix}$, then find the point of intersection of resulting lines.
5. Prove that under a 2x2 transformation matrix a pair of parallel lines is transformed to a pair of parallel lines.
6. If the line AB with slope m is transformed to the line A^*B^* with slope m^* using 2×2 transformation matrix $[T]=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$, then prove that $m^*=\dfrac{b+dm}{a+cm}$.
7. If the line $y=mx+k$ is transformed to the line $y^*=m^*x^*+k^*$ under a 2×2 transformation matrix $[T]=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$ then prove that $k^*=k\dfrac{(\mathrm{ad}-\mathrm{bc})}{a+cm}$.
8. If the line $y=4x+5$ is transformed by the transformation matrix $[T]=\begin{bmatrix}4 & -2\\ 1 & 3\end{bmatrix}$, then find the equation of the transformed line.
9. Rotate the ΔABC with vertices A[1, 2], B[3, 6] and C[3, 1] through an angle 90° about the point P[4, 8].
10. Rotate the line segment between the points A[1, 2] and B[3, 6] by an angle 90^0 about the midpoint of the line segment AB.
11. Reflect the ΔABC through the line y = 2x, where A[2, 4], B[0, 5] and C[-1, 1].
12. Reflect the ΔABC through the line x+y=0, where A[-2, 2], B[2, 3] and C[4, 5].
13. Reflect the ΔABC through the line x-2y+4=0, where A[2, 4], B[4, 6] and C[2, 6].
14. Prove that a 2x2 transformation matrix [T] preserves magnitude and angle between two vectors.
15. Find the combined transformation matrix [T] for the following sequence of transformations:
    (a) Reflection through the line x = 0;
    (b) Scaling in x – direction by 2 units.
    (c) Shearing in y – direction by -2 units.
    Apply it on object:

$$[X]=\begin{bmatrix}2 & -1\\ -2 & 1\end{bmatrix}$$

16. Find the concatenated transformation matrix [T] for the following sequence of transformations:
    a) Uniform scaling by 5 units.
    b) Translation in x and y directions by -2 and -7 units respectively.

c) Rotation about origin through 60^0.

17. Find the combined transformation matrix [T] for the following sequence of transformations:
    a) Scaling in x and y coordinates by 5 and 4 units respectively.
    b) Reflection through origin.
    c) Rotation about the origin through -90^0.

    Apply it on the object $[X]=\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$.
18. Obtain the combined transformation matrix for the following sequence of transformations:
    a) Rotation about origin through 30^0.
    b) Shearing in x and y directions by -3 and 4 units respectively.
    c) Translation in x and y directions by 1 and 2 units respectively.

    Apply it on the point P[6, -8].
19. Find the point of intersection at infinity for the lines:
    $2x+y=1,$
    $2x+y=2$

Ch.2 3D Transformation

2 Marks

1. Write the transformation matrix for translation in x and z directions by -3 and 5 units respectively.
2. Find the direction cosines of the unit vector perpendicular to the plane $x+2y+2z=0$.
3. Explain parallel projection.
4. Write the transformation matrix to create rear view of an object.
5. Define foreshortening factor.
6. Write the transformation matrix for perspective projection onto y=0 plane with centre of projection at [0,5,0].
7. What are the types of axonometric projection?

5 Marks

1. Write an algorithm to rotate an object [X] through an angle θ about an arbitrary axis passing through the point (x_0, y_0, z_0).

2. Rotate object $[X]=\begin{bmatrix}1 & 1 & 2 & 1\\ 2 & 1 & 2 & 1\end{bmatrix}$ about the local X axis passing through the point $\begin{bmatrix}\frac{3}{2} & \frac{3}{2} & \frac{3}{2}\end{bmatrix}$, through an angle $\theta = 30^0$.
3. Find the angles of rotation about x-axis and about y-axis so that the plane $x+2y+3z=0$ coincides with $z=0$ plane.
4. Obtain the Cabinet and Cavalier projections of the object $[X]=\begin{bmatrix}2 & 1 & -3\\ -4 & 2 & 6\end{bmatrix}$ with horizontal inclination angle $\alpha = 20^0$.
5. Find concatenated transformation matrix for the following sequence of transformations:
   a) Shearing in x-direction proportional to y and z coordinates by 2 and 3 units respectively.
   b) Translation in x, y and z directions by 10, 20 and 30 units respectively.
   c) Orthographic projection onto x=0 plane.

   Apply it on the origin O[0,0,0].
6. Find the combined transformation matrix [T] for the following sequence of transformations:
   a) Rotation about x-axis through 75^0.
   b) Translation in x, y and z directions by 4, 4 and 5 units respectively.
   c) Perspective transformation with centre of projection at $Z_c = 10$ on z-axis.
7. Obtain the Cavalier projection of the object $[X]=\begin{bmatrix}0 & 1 & 1\\ 1 & 0 & 1\\ 1 & 1 & 0\end{bmatrix}$ with horizontal inclination angle 35^0.
8. Develop the left side view and top view of the object $[X]=\begin{bmatrix}2 & 1 & 0\\ 0 & 2 & 1\\ 1 & 0 & 2\end{bmatrix}$.
9. Determine the diametric projection matrix if the foreshortening factor along z-axis is $\frac{2}{3}$ with $\phi > 0,\ \theta < 0$, where ϕ is the angle of rotation about y-axis and θ is the angle of rotation about x-axis.
10. Find the isometric projection of the line segment between the points A[-1, 2, 1] and B[3, -1, 6] with $\phi > 0,\ \theta > 0$.

Ch.3 Plane Curves

2 Marks

1. Determine the increment in angle θ to generate 10 equally spaced points on the ellipse $\frac{x^2}{100}+\frac{y^2}{400}=1$.
2. Determine the increment in parameter θ to generate 5 points on the parabolic segment $y^2=8x$ for $10\le y\le 20$.
3. Find the increment $\delta\theta$ to generate 8 points on the parabolic segment in the first quadrant for $8\le x\le 28$ where equation of parabola is $y^2=16x$.

5 Marks

1. Generate uniformly spaced 4 points on the arc of circle in the first quadrant, where the equation of circle is $x^2+y^2=4$.
2. Obtain uniformly spaced 6 points on the circle $x^2+y^2=9$.
3. Compute uniformly spaced 4 points on the arc of the ellipse $\frac{(x-1)^2}{16}+\frac{(y-2)^2}{9}=1$.
4. Compute uniformly spaced 4 points on the ellipse, $\frac{(x-1)^2}{25}+\frac{(y-2)^2}{16}=1$, inclined 45^0 with the horizontal.
5. Generate equally spaced 4 points on the parabolic segment in the first quadrant of parabola, $y^2=20x$, for $8\le y\le 12$.
6. Generate uniformly spaced 5 points on the hyperbolic segment in the first quadrant for $8\le x\le 12$, where equation of hyperbola is $\frac{x^2}{9}-\frac{y^2}{4}=1$.

Ch.4 Bezier Curve

2 Marks

1. Write any two properties of Bezier curve.
2. Write the matrix equation form of parametric equation of Bezier curve for three control points B_0, B_1, B_2.
3. State the general parametric equation of the Bezier curve and also obtain matrix representation of the cubic Bezier curve.

5 Marks

1. Write the parametric equation of the Bezier curve with the control points $B_0[-3, 1]$, $B_1[1, 4]$ and $B_2[5, -1]$. Hence find P(0.35), P(0.5) and P(0.65).
2. Obtain the parametric equation of the Bezier curve determined by the control points $B_0[1, 1]$, $B_1[1, 3]$, $B_2[3, 3]$ and $B_3[3, 1]$. Hence find P(0.25) and P(0.75).

Sub: Operation Research

Topic: Formulation of L.P.P

1. A firm manufactures three types of products A , B and C, for which the profits are Rs. 3, Rs 2 and Rs 4 respectively. The firm has two machines M1 and M2. The processing time for each machine is given below.

Product→ / Machine↓	A	B	C
M1	4	3	5
M2	2	2	4

Machine M1 and M2 have 2000 and 2500 minutes respectively. The firm must manufacture minimum 100 A's, 200 B's and 50 C's; but not more than 150 A's. Formulate the above problem as L.P.P to maximize the profit.

2. A company produces two products A and B. The sale volume for A is at least 80% of the total sale of A and B. However company cannot sell more than 100 units of A per day. Both products use one raw material whose maximum availability is limited to 300 Kg per day. The usage rates of the raw material are 3 Kg per unit of A and 5 Kg per unit of B. The unit profits of A and B are Rs. 200v and Rs. 500 respectively. Formulate this problem as L.P.P to maximize the profit.
3. A animal food company must produce 200 Kg of mixture consisting of ingredient A and B daily. The cost of ingredient A is Rs. 3 per Kg and that of ingredient B is Rs. 8 per Kg. Not more than 80 kg of A can be used, and at least 60 Kg of B must be used. Formulate the above problem as L.P.P
4. A paper mill produces two grades of papers namely X and Y. Because of raw material restrictions, it cannot produce more than 400 tons of grade X and 300 tons of grade Y in a week. There are 160 production hours in a week. It requires 0.2 and 0.4 hours to produce a ton of products X and Y respectively with corresponding profits of Rs. 200 and Rs. 500 per ton. Formulate the above problem as L.P.P
5. A firm can produce three types of clothes say A, B and C. The cloths are made of three colours of wools say, Red, Green and Blue. One unit of cloth A need 2 meters of Red wool and 3 meters of Blue wool; one unit of cloth B need 3 meters of Red, 2 meters of Green and 2 meters of Blue wool and one unit of cloth C need 5 meters of Green wool and 4meters of Blue wool. The firm has only a stock of 800 meters of Red wool, 1000 meters of Green wool and 1500 meters of Blue wool. Suppose that the profit per unit cloths of A, B and C are Rs.3, Rs. 4 and Rs.5 respectively. Determine how the firm should use the available material, so as to maximize the income from the finished clothes.
6. A company produces two types of presentation goods A and B that require gold and silver. Eanh unit of type A require 3gm of silver and 1 gm of gold while b requires 1 gm

of silver and 2 gm of gold. The company can produce 9 gm of silver and 8 gm of gold. If each unit of A brings a profit of Rs.40 and that of B Rs. 50, determine the number of units of each type that should be produced to maximize the profit.

7. A manufacture produces bicycles and tricycles each of which must be processed through two machines A and B. Machine A is available maximum of 130 hours and B is available maximum of 170 hours. Manufacturing a tricycle requires 5 hours of machine A and 3 hours of machine B. Manufacturing a bicycle requires 8 hours of machine A and 10 hours of machine B. If profits are Rs.55 for tricycle and Rs.75 for bicycle, formulate the problem as L.P.P to have maximum profit.

Graphical Method:

2 Marks:

1. Draw the feasible region for the following constraints:

$$x_1 + 3x_2 \geq 6$$
$$x_1 + x_2 \geq 5$$
$$x_1, x_2 \geq 0$$

2. Draw the graph and highlight the feasible region for the following constraints

$$x + 2y \geq 6$$
$$x + y \geq 9$$
$$x + y = 0$$
$$x, y \geq 0$$

3. Draw the feasible region for the following constraints:

$$x \geq 5$$
$$y \geq 3$$
$$x, y \geq 0$$

4. Draw the feasible region for the following constraints:

$$5x - y \geq 0$$
$$x + y = 6$$
$$x, y \geq 0$$

5. Show the feasible region for the following LPP

Maximize: $z = x_1 + x_2$

Subject to:

$$2x_1 + x_2 \leq 10$$
$$2x_1 + x_2 = 10$$
$$x_1, x_2 \geq 0$$

6. Draw the feasible region for the following constraints:

$$x+y\le 2$$
$$2x+y\ge 3$$
$$x,\ y\ge 0$$

5 Marks:

Q1. Solve the L.P.P by graphical method

Minimize: $Z=20x+10y$

$$x+2y\le 40$$
$$3x+y\ge 30$$
$$4x+3y\ge 60$$
$$x,y\ \ge 0$$

Q2. Solve the L.P.P by graphical method

Minimize: $Z=3x_1+9x_2$

$$x_1+x_2\ge 5$$
$$2x_1-3x_2\le 0$$
$$3x_1-x_2\ge 0$$
$$x_1,x_2\ge 0$$

Q3. Solve the L.P.P by graphical method

Maximize: $Z=5x_1+7x_2$

$$x_1+x_2\le 4$$
$$x_1+3x_2\le 24$$
$$10x_1+7x_2\ge 35$$
$$x_1,x_2\ge 0$$

Q4. Solve the L.P.P by graphical method

Maximize: $Z=3x+4y$

$$5x+4y\le 200$$
$$3x+5y\le 150$$
$$5x+4y\ge 100$$
$$x,y\ \ge 0$$

Q5. Solve the following LPP graphically

Maximize: $Z = 30x + 20y$

$$x + 2y \le 6$$
$$y \le 2$$
$$x - y \ge -1$$
$$x, y \ge 0$$

Topic: Simplex Method and Big-M Method

2 Marks:

Q1. Write two application of linear programming.

Q2. Define:

i. Surplus variable.

ii. Slack variable.

Q3. Write the following L.P.P in the standard form.

Maximize $Z = x_1 - x_2 + 2x_3$

Subject to:

$$x_1 - x_3 \ge 4$$
$$x_2 + 2x_3 \le 5$$
$$x_1 - x_2 + x_3 = 3$$
$$x_1, x_2, x_3 \ge 0$$

Q4. Write the following L.P.P in the standard form.

Minimize $Z = 4x_1 - x_2 + 3x_3$

Subject to:

$$2x_1 + 4x_3 \ge 5$$
$$x_1 + 3x_3 \ge 4$$
$$x_1 - x_2 + x_3 \le 7$$
$$x_1, x_2, x_3 \ge 0$$

Q5. Show that following LPP has unbounded solution

Maximize: $Z = 2x_1 + 3x_2$

Subject to:

$$x_1 \le 5$$
$$2x_1 - 3x_2 \le 6$$
$$x_1, x_2 \ge 0$$

Q6. Comment on the solution of the LPP, for which the last table of simplex method is given below. (R denotes Artificial variable)

C_B	C_J					-M	Const
	X_B	X_1	X_2	S_1	S_2	R	
2	X_2	2	1	1	0	0	2
-M	R	-5	0	-4	-1	1	4
Z_J - C_J		5M+1	0	4M+2	M	0	

5 Marks:

Q1. Solve the following LPP by simplex method

Maximize: $Z = 2x_1 + x_2$

Subject to:

$$2x_1 - x_2 \leq 1$$
$$x_1 - x_2 \geq 1$$
$$x_1, x_2 \geq 0$$

Q2. Solve the following LPP by simplex method

Maximize: $Z = x_1 + x_2$

Subject to:

$$-x_1 + 3x_2 \leq 10$$
$$x_1 + x_2 \leq 6$$
$$x_1 + x_2 \leq 2$$
$$x_1, x_2 \geq 0$$

Q3. Solve the following L.P.P by simplex method.

Maximize $Z = 2x_1 + 3x_2 + 4x_3$

Subject to:

$$3x_1 - 2x_3 \leq 41$$
$$2x_1 + x_2 + x_3 \leq 35$$
$$2x_1 + 3x_3 \leq 30$$
$$x_1, x_2, x_3 \geq 0$$

Q4. Solve the following L.P.P by simplex method.

Maximize $Z = 30x_1 + 16x_2 + 25x_3$

Subject to:

$$8x_1 + 4x_2 + 5x_3 \le 1000$$
$$5x_1 + 3x_2 + 3x_3 \le 650$$
$$9x_1 + 6x_2 + 9x_3 \le 1260$$
$$x_1, x_2, x_3 \ge 0$$

Q5. Solve the following L.P.P by simplex method.

Maximize $Z = 3x_1 + 5x_2 + 4x_3$

Subject to:

$$2x_1 + 3x_2 \le 8$$
$$2x_2 + 5x_3 \le 10$$
$$3x_1 + 2x_2 + 4x_3 \le 15$$
$$x_1, x_2, x_3 \ge 0$$

Q6. Solve the following L.P.P (Big M)

Minimize $Z = 4x_1 + 3x_2$

Subject to:

$$2x_1 + x_2 \ge 10$$
$$-3x_1 + 2x_2 \le 6$$
$$x_1 + x_2 \ge 6$$
$$x_1, x_2 \ge 0$$

Q7. Solve the following L.P.P (Big M)

Minimize $Z = 4x_1 + x_2$

Subject to:

$$3x_1 + x_2 = 3$$
$$4x_1 + 3x_2 \ge 6$$
$$x_1 + 2x_2 \le 4$$
$$x_1, x_2 \ge 0$$

Q8. Solve the following L.P.P by simplex method

Maximize $Z = 40x_1 + 35x_2$

Subject to:

$$3x_1 + x_2 = 3$$
$$4x_1 + 3x_2 \geq 6$$
$$x_1 + 2x_2 \leq 4$$
$$x_1, x_2 \geq 0$$

Q9. Consider the LPP

Maximize $Z = 2x_1 + 4x_2 + 4x_3 - 3x_4$

Subject to:

$$x_1 + x_2 + x_3 = 4$$
$$x_1 + 4x_2 + x_4 = 8$$
$$x_1, x_2, x_3, x_4 \geq 0$$

Solve the above LPP with x_3 *and* x_4 as the starting basic variables and without using an artificial variable.(x_3 *and* x_4 are the slack variable with non-zero objective coefficients.)

10 Marks:

Q1. Solve the following LPP by simplex method

Maximize $Z = 4x_1 + 5x_2 - 3x_3$

Subject to:

$$x_1 + x_2 + x_3 = 10$$
$$x_1 - x_2 \geq 1$$
$$2x_1 + 3x_2 + x_3 \leq 30$$
$$x_1, x_2, x_3 \geq 0$$

Topic: Dual

2 Marks

Q1. Write the dual of the following L.P.P

Maximize: $Z = 5x_1 - 2x_2 + 3x_3$

Subjected to:

$$2x_1 + 2x_2 - x_3 \geq 2$$
$$3x_1 - 4x_2 \leq 4$$
$$x_2 + 3x_3 \leq 5$$
$$x_1, x_2, x_3 \geq 0$$

Q2. Write the dual of the following L.P.P

Minimize $Z = 2x_1 + 10x_2 + 12x_3$

Subject to:

$x_1 + 5x_2 + 3x_3 \geq 5$

$x_1 + x_2 + 4x_3 \geq 4$

$x_1, x_2, x_3 \geq 0$

Q3. Write the dual of the following L.P.P

Maximize $Z = 5x_1 + 2x_2$

Subject to:

$4x_1 - x_2 \geq 4$

$x_1 + 2x_2 = 6$

$x_1 - 2x_2 \leq 3$

$x_1, x_2 \geq 0$

Q4. Write the dual of the following L.P.P

Maximize: $Z = 3x_1 + 2x_2$

Subject to:

$x_1 + x_2 \geq 1$

$x_1 + x_2 \leq 7$

$x_1 + 2x_2 \leq 10$

$x_2 \leq 3$

$x_1, x_2 \geq 0$

Q5. Write the dual of the following L.P.P

Maximize $Z = 5x_1 + 12x_2 + 4x_3$

Subject to:

$x_1 + 2x_2 + x_3 \leq 10$

$2x_1 - x_2 + 3x_3 = 8$

$x_1, x_2, x_3 \geq 0$

10 Marks:

Q1. Write the dual of the following L.P.P. Using simplex method solve this dual and hence write the solution of the following primal

Maximize: $Z = 3x_1 + 2x_2$

Subject to:

$$x_1 + x_2 \le 4$$
$$x_1 - x_2 \le 2$$
$$x_1, x_2 \ge 0$$

Q2. Write the dual of the following L.P.P.

Maximize: $Z = 5x_1 - 2x_2 + 3x_3$

Subject to:

$$2x_1 + 2x_2 - x_3 \ge 2$$
$$3x_1 - 4x_2 \le 3$$
$$x_2 + 3x_3 \le 5$$
$$x_1, x_2, x_3 \ge 0$$

Solve the dual problem using simple method, hence or otherwise find the solution of the primal problem

Topic: Transportation Problem

2 Marks:

Q1. Give the names of methods of finding an IBFS of a transportation problem. Which method gives better solution?

Q2. How the degeneracy is resolved in transportation problem?

Q3. When is the Transportation Problem is degenerate?

Q4. Use the North –West Corner rule to find an IBFS to the following Transportation Problem

	D1	**D2**	**D3**	**Supply**
O1	5	3	12	70
O2	3	10	4	70
Demand	30	65	45	

Q5. Obtain an IBFS for the following Transportation Problem by North –West Corner Method.

	D1	**D2**	**D3**	**Supply**
O1	5	3	7	30
O2	2	6	9	30
Demand	20	20	20	

Q6. Following is an IBFS of a Transportation Problem. Is the solution optimal? Justify

5	3 5	2 10
1 25	5	4 15

5 Marks:

Q1. Solve the following Assignment Problem to Maximize the Profit:

	J1	J2	J3	J4
M1	100	140	280	70
M2	130	160	200	60
M3	80	130	300	90
M4	150	110	250	50

Q2. Consider the following cost matrix of a transportation problem:

	D1	D2	D3	Supply
O1	5	1	7	10
O2	6	4	6	80
O3	3	2	5	15
Demand	75	20	50	

There is no enough supply therefore some demands may not be satisfied. Using penalty cost for unsatisfied demand units as 5, 3 and 1 for destination D1, D2 and D3 respectively, find an IBFS by Least Cost Method.

Q3. Obtain an IBFS for the following Transportation Problem using Vogel's Approximation Method:

	D1	D2	D3	D4	Supply
O1	7	2	10	4	70
O2	12	6	3	9	55
O3	14	13	5	5	90
Demand	85	35	50	45	

Q4. Show that the following solution of a Transportation Problem is an optimal solution. Also find the alternate optimal solution:

1 20	2	1 10	4
3	3 20	2 20	1 10
4	2 20	5	9

Q5. Find an IBFS of the following transportation problem by least cost method.

	W1	W2	W3	W4	Supply
F1	30	25	40	20	100
F2	29	26	35	40	250
F3	31	33	37	30	150
Demand	90	160	200	50	500

10 Marks

Q1. Find an IBFS of the following transportation problem by Vogel's Approximation Method and obtain the optimal solution by using MODI Method.

	D1	D2	D3	D4	Supply
O1	9	5	3	5	225
O2	9	10	13	7	75
O3	14	5	3	7	100
Demand	225	80	95	100	

Q2. Find an IBFS of the following transportation problem by Vogel's Approximation Method and obtain the optimal solution by using MODI Method.

	D1	D2	D3	D4	Supply
O1	75	39	48	57	23
O2	10	48	64	9	44
O3	0	50	24	30	33
Demand	23	31	16	30	

Q3. Find an IBFS of the following transportation problem by North-west corner Method and also find its optimal solution by using MODI Method.

	D1	D2	D3	Capacity
S1	2	2	3	10
S2	4	1	2	15
S3	1	3	1	40
Demand	20	15	30	

Q4. Following is the IBFS of the transportation problem. Is the solution optimal? Justify. If not find the optimal solution.

18	16	16 85	19	17 15
19	20 80	21	18 45	22
16 60	17	19	16 60	17 55

Q5. Find an IBFS of the following transportation problem by least cost entry method. Determine whether it is optimal solution Justify. If not find the optimal solution .

	D1	**D2**	**D3**	**D4**	**Supply**
O1	3	4	6	3	30
O2	3	5	7	0	50
O3	2	6	5	7	70
Demand	22	41	44	43	

Topic: Assignment Problem

2 Marks

Q1. How do we make an unbalanced Assignment problem balanced?

Q. Solve the following Assignment Problem

	I	**II**	**III**
A	10	5	9
B	4	8	5
C	3	4	1

Q2. Solve the following Assignment Problem

	I	**II**	**III**
A	5	2	1
B	4	3	3
C	3	4	4

Q3. Solve the following Assignment Problem

	I	**II**	**III**
A	8	6	7
B	15	10	5
C	4	13	6

Q4. Solve the following Assignment Problem

	I	II	III
A	15	10	9
B	9	15	10
C	10	12	8

Q5. Solve the following Assignment Problem

	I	II	III
A	2	1	3
B	5	2	1
C	3	4	9

5 Marks

Q1. Solve the following Assignment Problem

	A	B	C	D
I	45	40	51	67
II	57	42	63	55
III	49	52	48	64
IV	41	45	60	55

Q2. Solve the following Assignment Problem to maximize the profit:

	J1	J2	J3	J4
MI	100	140	280	70
MII	130	160	200	60
MIII	80	130	300	90
MIV	150	110	250	50

Q3. Solve the following Assignment Problem

	A	B	C	D
I	10	10	-	7
II	12	9	7	8
III	14	8	10	-
IV	12	7	11	12

Q4. Solve the following Assignment Problem

	I	II	III	IV	V
A	49	33	61	10	29
B	40	27	50	38	52
C	24	19	0	40	30
D	63	47	24	34	31
E	65	65	65	65	65

Q5. Solve the following Assignment Problem to minimize the total cost:

	I	II	III	IV	V
A	160	130	175	190	200
B	135	120	130	160	175
C	140	110	155	170	185
D	50	50	80	80	110
E	55	35	70	80	105

10 Marks:

Q1. Solve the following Assignment Problem

	I	II	III	IV	V
A	21	27	18	26	30
B	19	17	22	16	25
C	23	26	25	22	26
D	31	34	27	38	36
E	24	20	22	21	25

Q2. Solve the following Assignment Problem for maximization

	I	II	III	IV	V
A	32	38	40	28	40
B	40	24	28	21	36
C	41	27	33	30	37
D	22	38	41	36	36
E	29	33	40	35	39

Topic: Game Theory

2 marks:

Q1. Define:

i. Mixed strategy in game theory.

ii. Fair Game.

Q2. Define pure strategy in the game theory.

Q3. State the principle of dominance for rows in game theory.

Q4. Define “Two person zero sum game”.

Q5. Define the term “pay-off matrix”.

Q6. Find the saddle point for the following pay-off matrix.

$$\begin{bmatrix} 8 & 1 & 9 \\ 6 & 5 & 7 \\ 7 & 2 & 1 \end{bmatrix}$$

Q7. Find the saddle point for the following pay-off matrix.

$$\begin{bmatrix} 10 & 2 & 3 \\ 6 & 5 & 7 \\ -7 & 4 & 0 \end{bmatrix}$$

8. Solve the game, whose pay-off matrix is:

$$\begin{array}{cc} & \text{Player B} \\ \text{Player A} & \begin{bmatrix} 5 & 7 & 6 \\ 4 & 0 & 1 \\ 2 & 9 & 3 \end{bmatrix} \end{array}$$

Q9. Find the saddle point of the game:

$$\begin{bmatrix} 8 & -2 & 9 & -3 \\ 6 & 5 & 6 & 8 \\ -2 & 4 & -9 & 5 \end{bmatrix}$$

Q10. Is the following game Fair game? Justify.

$$\begin{bmatrix} -1 & 0 & -3 \\ 2 & 1 & 0 \\ -4 & 2 & -1 \end{bmatrix}$$

Q11. Two players A and B play the coin tossing game. Each player chooses head (H) or tail (T) unknown to each other simultaneously. If the choices match (HH or TT), player A receives Rs.1, otherwise A pays B Rs.1. Write the payoff matrix for player A.

5 Marks:

Q1. Solve the following game graphically

$$\begin{bmatrix} 1 & 3 & 11 \\ 8 & 5 & 2 \end{bmatrix}$$

Q2. Solve the following game by graphical method

$$\begin{bmatrix} 2 & 2 & 3 & -1 \\ 4 & 3 & 2 & 6 \end{bmatrix}$$

Q3. Solve the following game by principal of dominance.

$$\begin{bmatrix} 4 & 3 & 1 & 8 & 8 \\ 6 & 5 & 4 & 6 & 7 \\ 2 & 4 & 3 & 3 & 8 \\ 5 & 6 & 2 & 2 & 4 \end{bmatrix}$$

Q4. Solve the following game by dominance principal.

$$\begin{bmatrix} 3 & 5 & 4 & 9 & 6 \\ 5 & 6 & 3 & 7 & 8 \\ 8 & 7 & 9 & 8 & 7 \\ 4 & 4 & 8 & 5 & 3 \end{bmatrix}$$

Q5. Solve the following 5 x 2 game graphically

$$\begin{bmatrix} -4 & 3 \\ -7 & 1 \\ -2 & -4 \\ -5 & -2 \\ -1 & -6 \end{bmatrix}$$

10 Marks:

Q1. Solve the following game by dominance principal:

$$\begin{bmatrix} 3 & 5 & 4 & 9 & 6 \\ 5 & 6 & 3 & 7 & 8 \\ 8 & 7 & 9 & 8 & 7 \\ 9 & 2 & 8 & 5 & 3 \end{bmatrix}$$

Q2. Solve the following game graphically

$$\begin{bmatrix} 2 & 7 \\ 3 & 5 \\ 11 & 2 \end{bmatrix}$$

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